CHAPTER 1

MEASUREMENT

Despite the mathematical beauty of some of its most complex and abstract theories, physics is above all an experimental science. It is therefore critical that those who make precise measurements be able to agree on standards in which to express the results of those measurements, so that they can be communicated from one laboratory to another and verified.

In this chapter we begin our study of physics by introducing some of the basic units of physical quantities and the standards that have been accepted for their measurement. We consider the proper way to express the results of calculations and measurements, including the appropriate dimensions and number of significant figures. We discuss and illustrate the importance of paying attention to the dimensions of the quantities that appear in our equations. Later in the text, other basic units and many derived units are introduced as they are needed.

1-1 PHYSICAL QUANTITIES, STANDARDS, AND UNITS

The laws of physics are expressed in terms of many different quantities: mass, length, time, force, speed, density, resistance, temperature, luminous intensity, magnetic field strength, and many more. Each of these terms has a precise meaning, and they form part of the common language that physicists and other scientists use to communicate with one another—when a physicist uses a term such as "kinetic energy," all other physicists will immediately understand what is meant. Each of these terms also represents a quantity that can be measured in the laboratory, and just as there must be agreement on the meaning of these terms, there must also be agreement about the units used to express their values. Without such agreement, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Such comparisons require the development and acceptance of a set of *standards* for units of measurement. For example, if a measurement of length is quoted as 4.3 meters, it means that the measured length is 4.3 times as long as the value accepted for a standard length defined to be "one meter." If two laboratories base their measurements on the same accepted standard for the meter, then presumably their results can be easily compared. For this to be possible, the accepted standards must be *accessible* to those who need to calibrate their secondary standards, and they must be *invariable* to change with the passage of time or with changes in their environment (temperature, humidity, etc.).

Maintaining and developing standards for measurement is an active branch of science. In the United States, the National Institute of Standards and Technology* (NIST) has the primary responsibility for this development. However, it is also necessary to have wide international agreement about standards, which has been accomplished through a series of international meetings of the General Conference on Weights and Measures (known by their French acroynm

* See http://physics.nist.gov/cuu for information about NIST's role in maintaining standards.

CGPM) beginning in 1889; the twenty-first meeting was held in 1999.*

Fortunately, it is not necessary to establish a measurement standard for every physical quantity—some quantities can be regarded as fundamental, and the standards for other quantities can be derived from the fundamental ones. For example, length and time were once regarded as fundamental quantities with their individual established standards (respectively the meter and the second); the measurement standard for speed (= length/time) could then be derived in terms of those standards. However, in more recent years the speed of light has been measured to a precision exceeding that of the former standard meter; as a result, today we still use a fundamental standard for the second, but we define the standard for length (the meter) in terms of the speed of light and the second (see Section 1-4). This case illustrates how measurements of increasing precision can change the established standards and how rapidly such standards evolve. Since the publication of the first edition of this textbook, the precision of the standard unit for time (the second) has improved by more than a factor of 1000.

The basic problem therefore is to choose a system involving the smallest number of physical quantities as fundamental and to agree on accessible and invariable standards for their measurement. In the next sections of this chapter we discuss the internationally accepted system and some of its fundamental quantities.

1-2 THE INTERNATIONAL SYSTEM OF UNITS**

At its various meetings, the General Conference on Weights and Measures selected as *base units* the seven quantities displayed in Table 1-1. This is the basis of the International System of Units, abbreviated SI from the French *Le Système International d'Unités*. SI is the modern form of what is known generally as the metric system.

Throughout the book we give many examples of SI derived units, such as speed, force, and electric resistance, that follow from Table 1-1. For example, the SI unit of force, called the *newton* (abbreviation N), is defined in terms of the SI base units as

$$1\ \mathrm{N} = 1\ \mathrm{kg \cdot m/s^2},$$

as we shall make clear in Chapter 3.

If we express physical properties such as the output of a power plant or the time interval between two nuclear events in SI units, we often find very large or very small numbers. For convenience, the General Conference on Weights and Measures recommended the prefixes shown in Table 1-2.

TABLE 1-1 SI Base Units

	SI Unit	
Quantity	*Name*	*Symbol*
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of substance	mole	mol
Thermodynamic temperature	kelvin	K
Electric current	ampere	A
Luminous intensity	candela	cd

Thus we can write the output of a typical electrical power plant, 1.3×10^9 watts, as 1.3 gigawatts or 1.3 GW. Similarly, we can write a time interval of the size often encountered in nuclear physics, 2.35×10^{-9} seconds, as 2.35 nanoseconds or 2.35 ns. As Table 1-1 shows, the kilogram is the only SI base unit that *already* incorporates one of the prefixes displayed in Table 1-2. Thus 10^3 kg is *not* expressed as 1 kilokilogram; instead, $10^3\ \mathrm{kg} = 10^6\ \mathrm{g} = 1$ Mg (megagram).

To fortify Table 1-1 we need seven sets of operational procedures that tell us how to produce the seven SI base units in the laboratory. We explore those for time, length, and mass in the next three sections.

Two other major systems of units compete with the International System (SI). One is the Gaussian system, in terms of which much of the literature of physics is expressed. We do not use the Gaussian system in this book. Appendix G gives conversion factors to SI units.

The other is the British system, still in daily use in the United States. The basic units, in mechanics, are length (the foot), force (the pound), and time (the second). Again Appendix G gives conversion factors to SI units. We use SI units in this book, but we sometimes give the British equivalents, to help those who are unaccustomed to SI units to acquire more familiarity with them. The United States continues to be the only developed country that, so far, has not adopted SI as its official unit system. However, SI is standard in all U.S. government laboratories and in many industries, especially those involved in foreign trade. The loss of the *Mars Climate Orbiter* spacecraft in September 1999 has

TABLE 1-2 SI Prefixes[a]

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{24}	yotta-	Y	10^{-1}	deci-	d
10^{21}	zetta-	Z	10^{-2}	**centi-**	c
10^{18}	exa-	E	10^{-3}	**milli-**	m
10^{15}	peta-	P	10^{-6}	**micro-**	μ
10^{12}	tera-	T	10^{-9}	**nano-**	n
10^{9}	**giga-**	G	10^{-12}	**pico-**	p
10^{6}	**mega-**	M	10^{-15}	femto-	f
10^{3}	**kilo-**	k	10^{-18}	atto-	a
10^{2}	hecto-	h	10^{-21}	zepto-	z
10^{1}	deka-	da	10^{-24}	yocto-	y

[a] In all cases, the first syllable is accented, as in na′-no-me′-ter. Prefixes commonly used in this book are shown in boldface type.

* See http://www.bipm.fr for the recommendations of this conference.

** See "SI: The International System of Units," by Robert A. Nelson (American Association of Physics Teachers, 1981). The "official" U.S. guide to the SI system can be found in Special Publication 811 of the National Institute of Standards and Technology (1995 edition).

been traced to the fact that the manufacturer reported some of the *Orbiter*'s characteristics in British units, which the NASA navigation team mistakenly took to be SI units. Careful attention to units can be very important!

SAMPLE PROBLEM 1-1. Any physical quantity can be multiplied by 1 without changing its value. For example, 1 min = 60 s, so 1 = 60 s/1 min; similarly, 1 ft = 12 in., so 1 = 1 ft/12 in. Using appropriate conversion factors, find (*a*) the speed in meters per second equivalent to 55 miles per hour, and (*b*) the volume in cubic centimeters of a tank that holds 16 gallons of gasoline.

Solution (*a*) For our conversion factors, we need (see Appendix G) 1 mi = 1609 m (so that 1 = 1609 m/1 mi) and 1 h = 3600 s (so 1 = 1 h/3600 s). Thus

$$\text{speed} = 55\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \times \frac{1609\text{ m}}{1\,\cancel{\text{mi}}} \times \frac{1\,\cancel{\text{h}}}{3600\text{ s}} = 25\text{ m/s}.$$

(*b*) One fluid gallon is 231 cubic inches, and 1 in. = 2.54 cm. Thus

$$\text{volume} = 16\,\cancel{\text{gal}} \times \frac{231\,\cancel{\text{in.}}^3}{1\,\cancel{\text{gal}}} \times \left(\frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}}\right)^3 = 6.1 \times 10^4\text{ cm}^3.$$

Note in these two calculations how the unit conversion factors are inserted so that the unwanted units appear in one numerator and one denominator, and thus cancel.

1-3 THE STANDARD OF TIME

The measurement of time has two aspects. For civil and for some scientific purposes we want to know the time of day so that we can order events in sequence. In most scientific work we want to know how long an event lasts (the time interval). Thus any time standard must be able to answer the questions "At what time does it occur?" and "How long does it last?" Table 1-3 shows the range of time intervals that can be measured. They vary by a factor of about 10^{63}.

We can use any phenomenon that repeats itself as a measure of time. The measurement consists of counting the repetitions, including the fractions thereof. We could use an oscillating pendulum, a mass–spring system, or a quartz crystal, for example. Of the many repetitive phenomena in nature the rotation of the Earth on its axis, which determines the length of the day, was used as a time standard for centuries. One (mean solar) second was defined to be 1/86,400 of a (mean solar) day.

Quartz crystal clocks based on the electrically sustained periodic vibrations of a quartz crystal serve well as secondary time standards. A quartz clock can be calibrated against the rotating Earth by astronomical observations and used to measure time in the laboratory. The best of these have kept time with a precision of about 1 second in 200,000 years, but even this precision is not sufficient for the demands of modern science, technology, and commerce.

In 1967, the 13th General Conference on Weights and Measures adopted a definition of the second based on a characteristic frequency of the radiation emitted by a cesium atom. In particular, they stated that

TABLE 1-3 Some Measured Time Intervals[a]

Time Interval	*Seconds*
Lifetime of proton	$>10^{40}$
Half-life of double beta decay of ^{82}Se	3×10^{27}
Age of universe	5×10^{17}
Age of pyramid of Cheops	1×10^{11}
Human life expectancy (U.S.)	2×10^{9}
Time of Earth's orbit around the Sun (1 year)	3×10^{7}
Time of Earth's rotation about its axis (1 day)	9×10^{4}
Period of typical low-orbit Earth satellite	5×10^{3}
Time between normal heartbeats	8×10^{-1}
Period of concert-A tuning fork	2×10^{-3}
Period of oscillation of 3-cm microwaves	1×10^{-10}
Typical period of rotation of a molecule	1×10^{-12}
Shortest light pulse produced (1990)	6×10^{-15}
Lifetime of least stable particles	$<10^{-23}$

[a] Approximate values.

> *The second is the duration of 9,192,631,770 vibrations of a (specified) radiation emitted by a (specified) isotope of the cesium atom.*

Figure 1-1 shows the current national frequency standard, a so-called *cesium fountain clock* developed at the National Institute of Standards and Technology (NIST). Its precision is about 1 second in 20 million years.

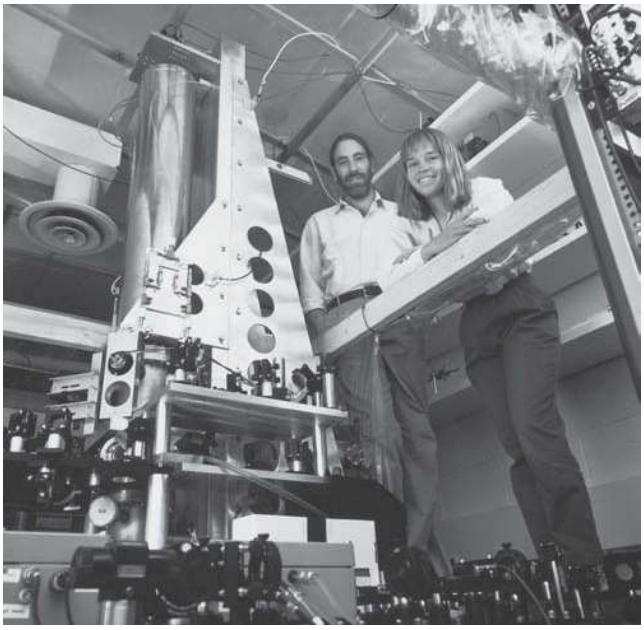

FIGURE 1-1. The National Frequency Standard NIST-F1, a so-called cesium fountain clock, developed at the National Institute of Standards and Technology. It is shown with its developers, Steve Jefferts and Dawn Meekhof. In this device extremely slow-moving cesium atoms are projected upward, covering a distance of about a meter before falling back under gravity to their launch position in about 1 second. Hence the label "fountain." The small speeds of these projected atoms make possible precise observation of the frequency of the atomic radiation that they emit. For more information, see http://www.nist.gov/public_affairs/releases/n99-22.htm.

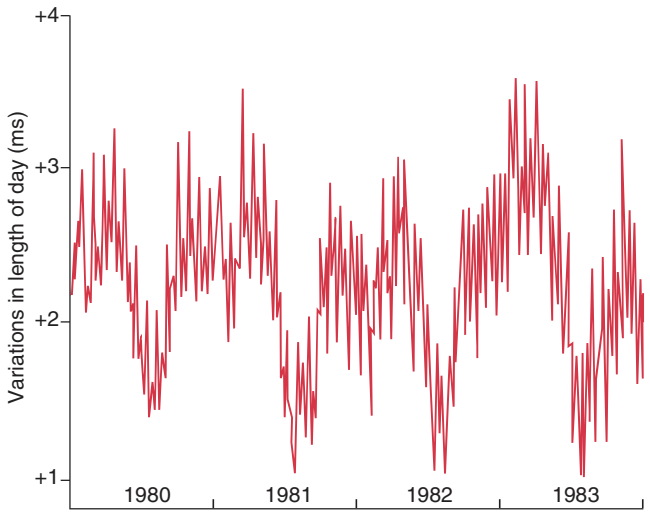

FIGURE 1-2. The variation in the length of the day over a 4-year period. Note that the vertical scale is only 3 ms = 0.003 s. See "The Earth's Rotation Rate," by John Wahr, *American Scientist,* January–February 1985, p. 41.

Cesium clocks housed in satellites form the basis of the Global Positioning System. Portable cesium clocks the size of a suitcase are commercially available. It is also possible to purchase desk-top clocks or wrist watches that, automatically and periodically updated by radio time signals from NIST, display "atomic time." Figure 1-2 shows, by comparison with a cesium clock, variations in the rate of rotation of the Earth over a 4-year period. These data suggest what a relatively poor time standard the Earth's rotation rate provides for precise work. Figure 1-3 shows the impressive record of improvements in time-keeping that have occurred over the past 300 years or so, starting with the invention of the pendulum clock by Christian Huygens in 1665.

The maintenance of timekeeping standards in the United States is the responsibility of the U.S. Naval Observatory (USNO) in Washington, DC. The USNO Master Clock represents the combined output of an assembly of cesium clocks and hydrogen masers housed in 20 separate, environmentally controlled vaults.*

1-4 THE STANDARD OF LENGTH**

The first international standard of length was a bar of a platinum–iridium alloy called the standard meter, which was kept at the International Bureau of Weights and Measures near Paris. The distance between two fine lines engraved near the ends of the bar, when the bar was held at a temperature of 0°C and supported mechanically in a prescribed way, was defined to be one meter. Historically, the meter was intended to be one ten-millionth of the distance from the north pole to the equator along the meridian line through Paris. However, accurate measurements showed that the standard meter bar differs slightly (about 0.023%) from this value.

Because the standard meter is not very accessible, accurate master copies of it were made and sent to standardized laboratories throughout the world. These secondary standards were used to calibrate other, still more accessible, measuring rods. Thus, until recently, every measuring rod or device derived its authority from the standard meter through a complicated chain of comparisons using micro-

* Information about time services provided by the USNO is available on the Internet at http://tycho.usno.navy.mil/ and by telephone at (202) 762-1401.

** See "The New Definition of the Meter," by P. Giacomo, *American Journal of Physics,* July 1984, p. 607.

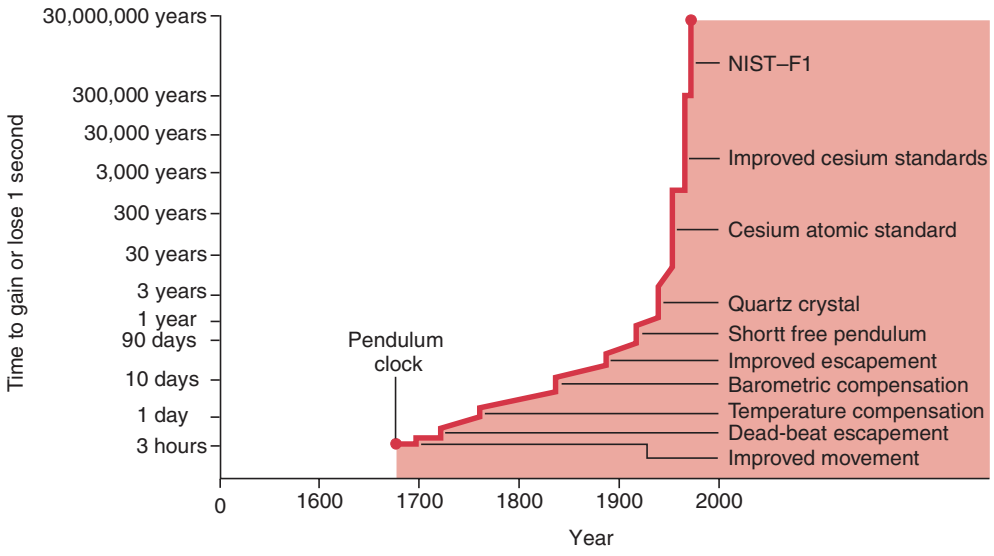

FIGURE 1-3. The improvement in timekeeping over the centuries. Early pendulum clocks gained or lost a second every few hours; present cesium clocks would do so only after several million years.

scopes and dividing engines. Since 1959 this statement had also been true for the yard, whose legal definition in the United States was adopted in that year to be

$$1 \text{ yard} = 0.9144 \text{ meter} \qquad \text{(exactly)},$$

which is equivalent to

$$1 \text{ inch} = 2.54 \text{ centimeters} \qquad \text{(exactly)}.$$

The accuracy with which the necessary intercomparisons of length can be made by the technique of comparing fine scratches using a microscope is no longer satisfactory for modern science and technology. A more precise and reproducible standard of length was obtained when the American physicist Albert A. Michelson in 1893 compared the length of the standard meter with the wavelength of the red light emitted by atoms of cadmium. Michelson carefully measured the length of the meter bar and found that the standard meter was equal to 1,553,163.5 of those wavelengths. Identical cadmium lamps could easily be obtained in any laboratory, and thus Michelson found a way for scientists around the world to have a precise standard of length without relying on the standard meter bar.

Despite this technological advance, the metal bar remained the official standard until 1960, when the 11th General Conference on Weights and Measures adopted an atomic standard for the meter. This standard was based on the wavelength in vacuum of a certain orange-red light emitted by atoms of the isotope of krypton with mass number 86, identified by the symbol ^{86}Kr.* Specifically, one meter was defined to be 1,650,763.73 wavelengths of this light. Using this standard, it became possible to compare lengths to a precision below 1 part in 10^9.

By 1983, the demands for higher precision had reached such a point that even the ^{86}Kr standard could not meet them and in that year a bold step was taken. The meter was redefined as the distance traveled by a light wave in a specified time interval. In the words of the 17th General Conference on Weights and Measures:

> *The meter is the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second.*

This is equivalent to saying that the speed of light c is now *defined* as

$$c = 299{,}792{,}458 \text{ m/s} \qquad \text{(exactly)}.$$

This new definition of the meter was necessary because measurements of the speed of light had become so precise that the reproducibility of the ^{86}Kr meter itself became the limiting factor. In view of this, it then made sense to adopt the speed of light as a defined quantity and to use it along with the precisely defined standard of time (the second) to redefine the meter.

Table 1-4 shows the range of measured lengths that can be compared with the standard.

TABLE 1-4 Some Measured Lengths[a]

Length	*Meters*
Distance to the farthest observed quasar	2×10^{26}
Distance to the Andromeda galaxy	2×10^{22}
Radius of our galaxy	6×10^{19}
Distance to the nearest star (Proxima Centauri)	4×10^{16}
Mean orbit radius for the most distant planet (Pluto)	6×10^{12}
Radius of the Sun	7×10^{8}
Radius of the Earth	6×10^{6}
Height of Mt. Everest	9×10^{3}
Height of a typical person	2×10^{0}
Thickness of a page in this book	1×10^{-4}
Size of a typical virus	1×10^{-6}
Radius of a hydrogen atom	5×10^{-11}
Effective radius of a proton	1×10^{-15}

[a] Approximate values.

SAMPLE PROBLEM 1-2. A light-year is a measure of length (not a measure of time) equal to the distance that light travels in 1 year. Compute the conversion factor between light-years and meters, and find the distance to the star Proxima Centauri (4.0×10^{16} m) in light-years.

Solution The conversion factor from years to seconds is

$$1 \text{ y} = 1 \text{ y} \times \frac{365.25 \text{ d}}{1 \text{ y}} \times \frac{24 \text{ h}}{1 \text{ d}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{60 \text{ s}}{1 \text{ min}}$$

$$= 3.16 \times 10^7 \text{ s}.$$

The speed of light is, to three significant figures, 3.00×10^8 m/s. Thus in 1 year, light travels a distance of

$$(3.00 \times 10^8 \text{ m/s})\,(3.16 \times 10^7 \text{ s}) = 9.48 \times 10^{15} \text{ m},$$

so that

$$1 \text{ light-year} = 9.48 \times 10^{15} \text{ m}.$$

The distance to Proxima Centauri is

$$(4.0 \times 10^{16} \text{ m}) \times \frac{1 \text{ light-year}}{9.48 \times 10^{15} \text{ m}} = 4.2 \text{ light-years}.$$

Light from Proxima Centauri thus takes about 4.2 years to travel to Earth.

1-5 THE STANDARD OF MASS

The SI standard of mass is a platinum–iridium cylinder kept at the International Bureau of Weights and Measures and assigned, by international agreement, a mass of 1 kilogram. Secondary standards are sent to standardizing laboratories in other countries and the masses of other

* The mass number is the total number of protons plus neutrons in the nucleus. Naturally occurring krypton has several different isotopes, corresponding to atoms with different mass numbers. It is important to specify a particular isotope for the standard, because the wavelength of the chosen radiation will vary from one isotope to another by about 1 part in 10^5, which is unacceptably large in comparison with the precision of the standard.

bodies can be found by an equal-arm balance technique to a precision of 1 part in 10^8.

The U.S. copy of the international standard of mass, known as Prototype Kilogram No. 20, is housed in a vault at the National Institute of Standards and Technology (see Fig. 1-4). It is removed no more than once a year for checking the values of tertiary standards. Since 1889 Prototype No. 20 has been taken to France twice for recomparison with the master kilogram. When it is removed from the vault two people are always present, one to carry the kilogram in a pair of forceps, the second to catch the kilogram if the first person should fall.

Table 1-5 shows some measured masses. Note that they vary by a factor of about 10^{83}. Most masses have been measured in terms of the standard kilogram by indirect methods. For example, we can measure the mass of the Earth (see Section 14-3) by measuring in the laboratory the gravitational force of attraction between two lead spheres and comparing it with the attraction of the Earth for a known mass. The masses of the spheres must be known by direct comparison with the standard kilogram.

On the atomic scale we have a second standard of mass, which is not an SI unit. It is the mass of the ^{12}C atom, which, by international agreement, has been assigned an atomic mass of 12 unified atomic mass units (abbreviation u), exactly and by definition. We can find the masses of other atoms to considerable accuracy by using a mass spectrometer (see Section 32-2). Table 1-6 shows some selected atomic masses, including the estimated uncertainties of measurement. We need a second standard of mass because present laboratory techniques permit us to compare atomic masses with each other to greater precision than we can presently compare them with the standard kilogram. However, development of an atomic mass standard to replace the standard kilogram is well under way. The relationship between the present atomic standard and the primary standard is approximately

$$1\ \text{u} = 1.661 \times 10^{-27}\ \text{kg}.$$

A related SI unit is the *mole,* which measures the quantity of a substance. One mole of ^{12}C atoms has a mass of exactly 12 grams and contains a number of atoms numerically equal to the Avogadro constant N_A:

$$N_A = 6.02214199 \times 10^{23}\ \text{per mole}.$$

This is an experimentally determined number, with an uncertainty of about one part in a million. One mole of any other substance contains the same number of elementary entities (atoms, molecules, or whatever). Thus 1 mole of helium gas contains N_A atoms of He, 1 mole of oxygen contains N_A molecules of O_2, and 1 mole of water contains N_A molecules of H_2O.

To relate an atomic unit of mass to a bulk unit, it is necessary to use the Avogadro constant. Replacing the standard kilogram with an atomic standard will require an improvement of at least two orders of magnitude in the precision of the measured value of N_A to obtain masses with precisions of 1 part in 10^8.

FIGURE 1-4. The National Standard Prototype Kilogram No. 20, resting in its double bell jar at the U.S. National Institute of Standards and Technology.

TABLE 1-5 Some Measured Masses[a]

Object	*Kilograms*
Known universe (estimate)	10^{53}
Our galaxy	2×10^{43}
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
Ocean liner	7×10^{7}
Elephant	4×10^{3}
Person	6×10^{1}
Grape	3×10^{-3}
Speck of dust	7×10^{-10}
Virus	1×10^{-15}
Penicillin molecule	5×10^{-17}
Uranium atom	4×10^{-26}
Proton	2×10^{-27}
Electron	9×10^{-31}

[a] Approximate values.

TABLE 1-6 Some Measured Atomic Masses

Isotope	*Mass (u)*	*Uncertainty (u)*
^{1}H	1.00782503214	0.00000000035
^{12}C	12.00000000	(exact)
^{64}Cu	63.9297679	0.0000015
^{109}Ag	108.9047551	0.0000032
^{137}Cs	136.9070836	0.0000032
^{208}Pb	207.9766358	0.0000031
^{238}Pu	238.0495534	0.0000022

1-6 PRECISION AND SIGNIFICANT FIGURES

As we improve the quality of our measuring instruments and the sophistication of our techniques, we can carry out experiments at ever increasing levels of precision; that is, we can extend the measured results to more and more *significant figures* and correspondingly reduce the *experimental uncertainty* of the result. Both the number of significant figures and the uncertainty tell something about our estimate of the precision of the result. That is, the result $x = 3$ m implies that we know less about x than the value $x = 3.14159$ m. When we declare $x = 3$ m, we mean that we are reasonably certain that x lies between 2 m and 4 m, whereas expressing x as 3.14159 m means that x probably lies between 3.14158 m and 3.14160 m. If you express x as 3 m when in fact you really believe that x is 3.14159 m, you are withholding information that might be important. On the other hand, if you express $x = 3.14159$ m when you really have no basis for knowing anything other than $x = 3$ m, you are being somewhat dishonest by claiming to have more information than you really do. Attention to significant figures is important when presenting the results of measurements and calculations, and it is equally as wrong to include too many as too few.

There are a few simple rules to follow in deciding how many significant figures to keep:

Rule 1. Counting from the left and ignoring leading zeros, keep all digits up to the first doubtful one. That is, $x = 3$ m has only one significant figure, and expressing this value as $x = 0.003$ km does not change the number of significant figures. If we instead wrote $x = 3.0$ m (or, equivalently, $x = 0.0030$ km), we would imply that we know the value of x to two significant figures. In particular, don't write down all 9 or 10 digits of your calculator display if they are not justified by the precision of the input data! Most calculations in this text are done with two or three significant figures.

Be careful about ambiguous notations: $x = 300$ m does not indicate whether there are one, two, or three significant figures; we don't know whether the zeros are carrying information or merely serving as place holders. Instead, we should write $x = 3 \times 10^2$ or 3.0×10^2 or 3.00×10^2 to specify the precision more clearly.

Rule 2. When multiplying or dividing, the number of significant figures in the product or quotient should be no greater than the number of significant figures in the least precise of the factors. Thus

$$2.3 \times 3.14159 = 7.2.$$

A bit of good judgment is occasionally necessary when applying this rule:

$$9.8 \times 1.03 = 10.1$$

because, even though 9.8 technically has only two significant figures, it is very close to being a number with three significant figures. The product should therefore be expressed with three significant figures.

Rule 3. In adding or subtracting, the least significant digit of the sum or difference occupies the same relative position as the least significant digit of the quantities being added or subtracted. In this case the *number* of significant figures is not important; it is the *position* that matters. For example, suppose we wish to find the total mass of three objects as follows:

$$\begin{array}{rl} 103.\mathbf{9} & \text{kg} \\ 2.1\mathbf{0} & \text{kg} \\ 0.31\mathbf{9} & \text{kg} \\ \hline 106.\mathbf{319} & \text{or} \quad 106.\mathbf{3}\ \text{kg} \end{array}$$

The least significant or first doubtful digit is shown in **boldface.** By rule 1, we should include only one doubtful digit; thus the result should be expressed as 106.3 kg, for if the "3" is doubtful, then the following "19" gives no information and is useless.

SAMPLE PROBLEM 1-3. You wish to weigh your pet cat, but all you have available is an ordinary home platform scale. It is a digital scale, which displays your weight in a whole number of pounds. You therefore use the following scheme: you determine your own weight to be 119 lbs, and then holding the cat you find your combined weight to be 128 lbs. What is the fractional or percentage uncertainty in your weight and in the weight of your cat?

Solution The least significant digit is the units digit, and so your weight is uncertain by about 1 pound. That is, your scale would read 119 lb for any weight between 118.5 and 119.5 lb. The fractional uncertainty is therefore

$$\frac{1\ \text{lb}}{119\ \text{lb}} = 0.008 \qquad \text{or} \qquad 0.8\%.$$

The weight of the cat is 128 lb − 119 lb = 9 lb. However, the uncertainty in the cat's weight is still about 1 lb, and so the fractional uncertainty is

$$\frac{1\ \text{lb}}{9\ \text{lb}} = 0.11 = 11\%.$$

Although the *absolute* uncertainty in your weight and the cat's weight is the same (1 lb), the *relative* uncertainty in your weight is an order of magnitude smaller than the relative uncertainty in the cat's weight. If you tried to weigh a 1-lb kitten by this method, the relative uncertainty in its weight would be 100%. This illustrates a commonly occurring danger in the subtraction of two numbers that are nearly equal: the relative or percentage uncertainty in the difference can be very large.

1-7 DIMENSIONAL ANALYSIS

Associated with every measured or calculated quantity is a *dimension.* For example, both the absorption of sound by an enclosure and the probability for nuclear reactions to occur have the dimensions of an area. The units in which the

quantities are expressed do not affect the dimension of the quantities: an area is still an area whether it is expressed in m^2 or ft^2 or acres or sabins (sound absorption) or barns (nuclear reactions).

Just as we defined our measurement standards earlier in this chapter as fundamental quantities, we can choose a set of fundamental dimensions based on independent measurement standards. For mechanical quantities, mass, length, and time are elementary and independent, so they can serve as fundamental dimensions. They are represented respectively by M, L, and T.

Any equation must be *dimensionally consistent;* that is, the dimensions on both sides must be the same. Attention to dimensions can often keep you from making errors in writing equations. For example, the distance x covered in a time t by an object starting from rest and moving subject to a constant acceleration a will be shown in the next chapter to be $x = \frac{1}{2}at^2$. Acceleration is measured in units such as m/s^2. We use square brackets [ ] to denote "the dimension of," so that $[x] = L$ or $[t] = T$. It follows that $[a] = L/T^2$ or LT^{-2}. Keeping the units, and therefore the dimension, of acceleration in mind, you will therefore never be tempted to write $x = \frac{1}{2}at$ or $x = \frac{1}{2}at^3$.

The analysis of dimensions can often help in working out equations. The following two sample problems illustrate this procedure.

SAMPLE PROBLEM 1-4. To keep an object moving in a circle at constant speed requires a force called the "centripetal force." (Circular motion is discussed in Chapter 4.) Do a dimensional analysis of the centripetal force.

Solution We begin by asking "On which mechanical variables could the centripetal force F depend?" The moving object has only three properties that are likely to be important: its mass m, its speed v, and the radius r of its circular path. The centripetal force F must therefore be given, apart from any dimensionless constants, by an equation of the form

$$F \propto m^a v^b r^c,$$

where the symbol $\propto$ means "is proportional to" and where a, b, and c are numerical exponents to be determined from analyzing the dimensions. As we wrote in Section 1-2 (and as we shall discuss in Chapter 3), force has units of $kg \cdot m/s^2$, and so its dimensions are $[F] = MLT^{-2}$. We can therefore write the centripetal force equation in terms of dimensions as

$$\begin{aligned} [F] &= [m^a]\,[v^b]\,[r^c] \\ MLT^{-2} &= M^a\,(L/T)^b\,L^c \\ &= M^a\,L^{b+c}\,T^{-b}. \end{aligned}$$

Dimensional consistency means that the fundamental dimensions must be the same on each side. Thus, equating the exponents,

exponents of M: $a = 1$;

exponents of T: $b = 2$;

exponents of L: $b + c = 1$, so $c = -1$.

The resulting expression is

$$F \propto \frac{mv^2}{r}.$$

The actual expression for centripetal force, derived from Newton's laws and the geometry of circular motion, is $F = mv^2/r$. The dimensional analysis gives us the exact dependence on the mechanical variables! This is really a happy accident, because dimensional analysis can't tell us anything about constants that do not have dimensions. In this case the constant happens to be 1.

SAMPLE PROBLEM 1-5. An important milestone in the evolution of the universe just after the Big Bang is the Planck time t_P, the value of which depends on three fundamental constants: (1) the speed of light (the fundamental constant of relativity), $c = 3.00 \times 10^8$ m/s; (2) Newton's gravitational constant (the fundamental constant of gravity), $G = 6.67 \times 10^{-11}\ m^3/s^2 \cdot kg$; and (3) Planck's constant (the fundamental constant of quantum mechanics), $h = 6.63 \times 10^{-34}\ kg \cdot m^2/s$. Based on a dimensional analysis, find the value of the Planck time.

Solution Using the units given for the three constants, we can obtain their dimensions:

$$\begin{aligned} [c] &= [m/s] = LT^{-1} \\ [G] &= [m^3/s^2 \cdot kg] = L^3T^{-2}M^{-1} \\ [h] &= [kg \cdot m^2/s] = ML^2T^{-1} \end{aligned}$$

Let the Planck time depend on these constants as

$$t_P \propto c^i\,G^j\,h^k,$$

where i, j, and k are exponents to be determined. The dimensions of this expression are

$$\begin{aligned} [t_P] &= [c^i]\,[G^j]\,[h^k] \\ T &= (LT^{-1})^i\,(L^3T^{-2}M^{-1})^j\,(ML^2T^{-1})^k \\ &= L^{i+3j+2k}\,T^{-i-2j-k}\,M^{-j+k}. \end{aligned}$$

Equating powers on both sides gives

exponents of L: $0 = i + 3j + 2k$

exponents of T: $1 = -i - 2j - k$

exponents of M: $0 = -j + k$

and solving these three equations for the three unknowns, we find

$$i = -\tfrac{5}{2}, \qquad j = \tfrac{1}{2}, \qquad k = \tfrac{1}{2}.$$

Thus

$$\begin{aligned} t_P &\propto c^{-5/2}G^{1/2}h^{1/2} \\ &= \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{(6.67 \times 10^{-11}\ m^3/s^2 \cdot kg)(6.63 \times 10^{-34}\ kg \cdot m^2/s)}{(3.00 \times 10^8\ m/s)^5}} \\ &= 1.35 \times 10^{-45}\ s. \end{aligned}$$

As commonly defined, the Planck time differs from this value by a factor of $(2\pi)^{-1/2}$. Such dimensionless factors cannot be found by this technique.

In similar fashion, we can determine the Planck length and the Planck mass, which also have very fundamental interpretations (see Exercises 32 and 33).

MULTIPLE CHOICE

1-1 Physical Quantities, Standards, and Units

1-2 The International System of Units

1-3 The Standard of Time

1-4 The Standard of Length

1-5 The Standard of Mass

1-6 Precision and Significant Figures

1. A student is calculating the surface area of a single sheet of paper. He measures the length to be $l = 27.9$ cm; he measures the width to be $w = 21.6$ cm. The student should record the area of the paper as
(A) 602.64 cm^2. (B) 602.6 cm^2.
(C) 602 cm^2. (D) 603 cm^2.

2. A student is calculating the thickness of a single sheet of paper. She measures the thickness of a stack of 80 sheets with vernier calipers, and finds the thickness to be $l = 1.27$ cm. To calculate the thickness of a single sheet she divides. Which of the following answers has the correct number of significant digits?
(A) 0.15875 mm. (B) 0.159 mm.
(C) 0.16 mm. (D) 0.2 mm.

1-7 Dimensional Analysis

3. The period of oscillation of a nonlinear oscillator depends on the mass m, with dimensions of M; a restoring force constant k with dimensions of $ML^{-2}T^{-2}$, and the amplitude A, with dimensions of L. Dimensional analysis shows that the period of oscillation should be proportional to
(A) $A\sqrt{m/k}$. (B) A^2m/k.
(C) $A^{-1}\sqrt{m/k}$. (D) A^2k^3/m.

QUESTIONS

1. How would you criticize this statement: "Once you have picked a standard, by the very meaning of 'standard' it is invariable"?

2. List characteristics other than accessibility and invariability that you would consider desirable for a physical standard.

3. Can you imagine a system of base units (Table 1-1) in which time was not included?

4. Of the seven base units listed in Table 1-1, only one—the kilogram—has a prefix (see Table 1-2). Would it be wise to redefine the mass of that platinum–iridium cylinder at the International Bureau of Weights and Measures as 1 g rather than 1 kg?

5. What does the prefix "micro-" signify in the words "microwave oven"? It has been proposed that food that has been irradiated by gamma rays to lengthen its shelf life be marked "picowaved." What do you suppose that means?

6. Many capable investigators, on the evidence, believe in the reality of extrasensory perception. Assuming that ESP is indeed a fact of nature, what physical quantity or quantities would you seek to define to describe this phenomenon quantitatively?

7. Name several repetitive phenomena occurring in nature that could serve as reasonable time standards.

8. You could define "1 second" to be one pulse beat of the current president of the American Association of Physics Teachers. Galileo used his pulse as a timing device in some of his work. Why is a definition based on the atomic clock better?

9. What criteria should be satisfied by a good clock?

10. Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on the successive days of a week the clocks read as follows:

Clock	Sun.	Mon.	Tues.	Wed.
A	12:36:40	12:36:56	12:37:12	12:37:27
B	11:59:59	12:00:02	11:59:57	12:00:07
C	15:50:45	15:51:43	15:52:41	15:53:39
D	12:03:59	12:02:52	12:01:45	12:00:38
E	12:03:59	12:02:49	12:01:54	12:01:52

Clock	Thurs.	Fri.	Sat.
A	12:37:44	12:37:59	12:38:14
B	12:00:02	11:59:56	12:00:03
C	15:54:37	15:55:35	15:56:33
D	11:59:31	11:58:24	11:57:17
E	12:01:32	12:01:22	12:01:12

How would you arrange these five clocks in the order of their relative value as good timekeepers? Justify your choice.

11. From what you know about pendulums, cite the drawbacks to using the period of a pendulum as a time standard.

12. How did Galileo know that the pendulum swings at the same frequency regardless of the amplitude? Note: since pendulums were crucial to the building of the first clocks, Galileo couldn't have used a clock to find the answer!

13. What is the uncertainty in a good sand-based egg timer? What about an hourglass? What about the candles used for marking time at night?

14. On June 30, 1981, the minute extending from 10:59 to 11:00 A.M. was arbitrarily lengthened to contain 61 s. The last day of 1989 also was lengthened by 1 s. Such a *leap second* is occasionally introduced to compensate for the fact that, as measured by our atomic time standard, Earth's rotation rate is slowly decreasing. Why is it desirable to readjust our clocks in this way?

15. A radio station advertises that it is "at 89.5 on your FM dial." What does this number mean?

16. Why are there no SI base units for area or volume?

17. The meter was originally intended to be one ten-millionth of the meridian line from the north pole to the equator that passes through Paris. This definition disagrees with the meter bar (adopted at that early date as a standard) by 0.023%. Does this mean that the standard meter bar is inaccurate to this extent?

18. The original meter's definition did *not* involve directly measuring the entire distance from the north pole to the equator. How was it done? Discuss any experimental uncertainty.

19. Can length be measured along a curved line? If so, how?

20. When the meter bar was taken to be the standard of length, its temperature was specified. Can length be called a fundamental property if another physical quantity, such as temperature, must be specified in choosing a standard?

21. In redefining the meter in terms of the speed of light, why did the delegates to the 1983 General Conference on Weights and Measures not simplify matters by defining the speed of light to be 3×10^8 m/s exactly? For that matter, why did they not define it to be 1 m/s exactly? Were both of these possibilities open to them? If so, why did they reject them?

22. The kilogram was originally defined so that the density of water was 1000 in metric units. Can a "metric" version of $\pi = 3.1415926535\ldots$ be redefined so that it is exactly equal to 22/7? What about saving considerable computation trouble and defining it as $\pi = 3$?

23. Suggest a way to measure (*a*) the radius of the Earth, (*b*) the distance between the Sun and the Earth, and (*c*) the radius of the Sun.

24. Suggest a way to measure (*a*) the thickness of a sheet of paper, (*b*) the thickness of a soap bubble film, and (*c*) the diameter of an atom.

25. If someone told you that every dimension of every object had shrunk to half its former value overnight, how could you refute this statement?

26. Is the current standard kilogram of mass accessible, invariable, reproducible, and indestructible? Does it have simplicity for comparison purposes? Would an atomic standard be better in any respect? Why don't we adopt an atomic standard, as we do for length and time?

27. Why do we find it useful to have two standards of mass, the kilogram and the ^{12}C atom?

28. How does one obtain the relation between the mass of the standard kilogram and the mass of the ^{12}C atom?

29. Suggest practical ways by which one could determine the masses of the various objects listed in Table 1-5.

30. Suggest objects whose masses would fall in the wide range in Table 1-5 between that of an ocean liner and the Moon, and estimate their masses.

31. Critics of the metric system often cloud the issue by saying things such as: "Instead of buying 1 lb of butter you will have to ask for 0.454 kg of butter." The implication is that life would be more complicated. How might you refute this?

EXERCISES

1-1 Physical Quantities, Standards, and Units

1-2 The International System of Units

1. Use the prefixes in Table 1-2 to express (*a*) 10^6 phones, (*b*) 10^{-6} phones, (*c*) 10^1 cards, (*d*) 10^9 lows, (*e*) 10^{12} bulls, (*f*) 10^{-1} mates, (*g*) 10^{-2} pedes, (*h*) 10^{-9} Nannettes, (*i*) 10^{-12} boos, (*j*) 10^{-18} boys, (*k*) 2×10^2 withits, (*l*) 2×10^3 mockingbirds. Now that you have the idea, invent a few more similar expressions. (See p. 61 of *A Random Walk in Science,* compiled by R. L. Weber; Crane, Russak & Co., New York, 1974.)

2. Some of the prefixes of the SI units have crept into everyday language. (*a*) What is the weekly equivalent of an annual salary of 36K (= 36 k$)? (*b*) A lottery awards 10 megabucks as the top prize, payable over 20 years. How much is received in each monthly check? (*c*) The hard disk of a computer has a capacity of 30 GB (= 30 gigabytes). At 8 bytes/word, how many words can it store?

1-3 The Standard of Time

3. Enrico Fermi once pointed out that a standard lecture period (50 min) is close to 1 microcentury. How long is a microcentury in minutes, and what is the percentage difference from Fermi's approximation?

4. New York and Los Angeles are about 3000 mi apart; the time difference between these two cities is 3 h. Calculate the circumference of the Earth.

5. A convenient substitution for the number of seconds in a year is π times 10^7. To within what percentage error is this correct?

6. (*a*) A unit of time sometimes used in microscopic physics is the *shake.* One shake equals 10^{-8} s. Are there more shakes in a second than there are seconds in a year? (*b*) Humans have existed for about 10^6 years, whereas the universe is about 10^{10} years old. If the age of the universe is taken to be 1 day, for how many seconds have humans existed?

7. In two different track meets, the winners of the mile race ran their races in 3 min 58.05 s and 3 min 58.20 s. In order to conclude that the runner with the shorter time was indeed faster, what is the maximum tolerable error, in feet, in laying out the distances?

8. A certain pendulum clock (with a 12-h dial) happens to gain 1 min/day. After setting the clock to the correct time, how long must one wait until it again indicates the correct time?

9. The age of the universe is about 5×10^{17} s; the shortest light pulse produced in a laboratory (1990) lasted for only 6×10^{-15} s (see Table 1-3). Identify a physically meaningful time interval approximately halfway between these two on a logarithmic scale.

10. Assuming that the length of the day uniformly increases by 0.001 s in a century, calculate the cumulative effect on the measure of time over 20 centuries. Such a slowing down of the Earth's rotation is indicated by observations of the occurrences of solar eclipses during this period.

11. The time it takes the Moon to return to a given position as seen against the background of fixed stars, 27.3 days, is called a sidereal month. The time interval between identical phases of the Moon is called a lunar month. The lunar month is longer than a sidereal month. Why and by how much?

1-4 The Standard of Length

12. Your French pen pal Pierre writes to say that he is 1.9 m tall. What is his height in British units?

13. (*a*) In track meets both 100 yards and 100 meters are used as distances for dashes. Which is longer? By how many meters is it longer? By how many feet? (*b*) Track and field records are kept for the mile and the so-called metric mile (1500 meters). Compare these distances.

14. The stability of the cesium clock used as an atomic time standard is such that two cesium clocks would gain or lose 1 s with respect to each other in about 300,000 y. If this same precision were applied to the distance between New York and San Francisco (2572 mi), by how much would successive measurements of this distance tend to differ?

15. Antarctica is roughly semicircular in shape with a radius of 2000 km. The average thickness of the ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of the Earth.)

16. A unit of area, often used in expressing areas of land, is the *hectare,* defined as 10^4 m^2. An open-pit coal mine consumes 77 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, is removed in this time?

17. Earth is approximately a sphere of radius 6.37×10^6 m. (*a*) What is its circumference in kilometers? (*b*) What is its surface area in square kilometers? (*c*) What is its volume in cubic kilometers?

18. The approximate maximum speeds of various animals follow, but in different units of speed. Convert these data to m/s, and thereby arrange the animals in order of increasing maximum speed: squirrel, 19 km/h; rabbit, 30 knots; snail, 0.030 mi/h; spider, 1.8 ft/s; cheetah, 1.9 km/min; human, 1000 cm/s; fox, 1100 m/min; lion, 1900 km/day.

19. A certain spaceship has a speed of 19,200 mi/h. What is its speed in light-years per century?

20. A new car is equipped with a "real-time" dashboard display of fuel consumption. A switch permits the driver to toggle back and forth between British units and SI units. However, the British display shows mi/gal while the SI version is the inverse, L/km. What SI reading corresponds to 30.0 mi/gal?

21. Astronomical distances are so large compared to terrestrial ones that much larger units of length are used for easy comprehension of the relative distances of astronomical objects. An *astronomical unit* (AU) is equal to the average distance from Earth to the Sun, 1.50×10^8 km. A *parsec* (pc) is the distance at which 1 AU would subtend an angle of 1 second of arc. A *light-year* (ly) is the distance that light, traveling through a vacuum with a speed of 3.00×10^5 km/s, would cover in 1 year. (*a*) Express the distance from Earth to the Sun in parsecs and in light-years. (*b*) Express a light-year and a parsec in kilometers. Although the light-year is much used in popular writing, the parsec is the unit preferred by astronomers.

22. The effective radius of a proton is about 1×10^{-15} m; the radius of the observable universe (given by the distance to the farthest observable quasar) is 2×10^{26} m (see Table 1-4). Identify a physically meaningful distance that is approximately halfway between these two extremes on a logarithmic scale.

1-5 The Standard of Mass

23. Using conversions and data in the chapter, determine the number of hydrogen atoms required to obtain 1.00 kg of hydrogen.

24. One molecule of water (H_2O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u. (*a*) What is the mass in kilograms of one molecule of water? (*b*) How many molecules of water are in the oceans of the world? The oceans have a total mass of 1.4×10^{21} kg.

25. In continental Europe, one "pound" is half a kilogram. Which is the better buy: one Paris pound of coffee for $9.00 or one New York pound of coffee for $7.20?

26. A room has dimensions of 21 ft × 13 ft × 12 ft. What is the mass of the air it contains? The density of air at room temperature and normal atmospheric pressure is 1.21 kg/m^3.

27. A typical sugar cube has an edge length of 1 cm. If you had a cubical box that contained 1 mole of sugar cubes, what would its edge length be?

28. A person on a diet loses 0.23 kg (corresponding to about 0.5 lb) per week. Express the mass loss rate in milligrams per second.

1-6 Precision and Significant Figures

29. For the period 1960–1983, the meter was defined to be 1,650,763.73 wavelengths of a certain orange-red light emitted by krypton atoms. Compute the distance in nanometers corresponding to one wavelength. Express your result using the proper number of significant figures.

30. (*a*) Evaluate 37.76 + 0.132 to the correct number of significant figures. (*b*) Evaluate 16.264 − 16.26325 to the correct number of significant figures.

1-7 Dimensional Analysis

31. Porous rock through which groundwater can move is called an aquifer. The volume V of water that, in time t, moves through a cross section of area A of the aquifer is given by

$$V/t = KAH/L,$$

where H is the vertical drop of the aquifer over the horizontal distance L; see Fig. 1-5. This relation is called Darcy's law. The quantity K is the hydraulic conductivity of the aquifer. What are the SI units of K?

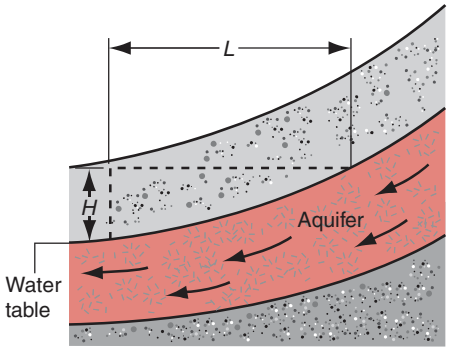

FIGURE 1-5. Exercise 31.

32. In Sample Problem 1-5, the constants h, G, and c were combined to obtain a quantity with the dimensions of time. Repeat the derivation to obtain a quantity with the dimensions of length, and evaluate the result numerically. Ignore any dimensionless constants. This is the Planck length, the size of the observable universe at the Planck time.

33. Repeat the procedure of Exercise 32 to obtain a quantity with the dimensions of mass. This gives the Planck mass, the mass of the observable universe at the Planck time.

PROBLEMS

1. Shortly after the French Revolution, as part of their introduction of the metric system, the revolutionary National Convention made an attempt to introduce decimal time. In this plan, which was not successful, the day—starting at midnight—was divided into 10 decimal hours consisting of 100 decimal minutes each. The hands of a surviving decimal pocket watch are stopped at 8 decimal hours, 22.8 decimal minutes. What time is it? See Fig. 1-6.

FIGURE 1-6. Problem 1.

2. The average distance of the Sun from the Earth is 390 times the average distance of the Moon from the Earth. Now consider a total eclipse of the Sun (Moon between Earth and Sun; see Fig. 1-7) and calculate (*a*) the ratio of the Sun's diameter to the Moon's diameter, and (*b*) the ratio of the Sun's volume to the Moon's volume. (*c*) The angle intercepted at the eye by the Moon is 0.52° and the distance between the Earth and the Moon is 3.82×10^5 km. Calculate the diameter of the Moon.

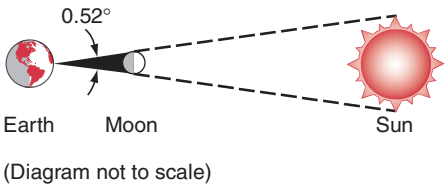

FIGURE 1-7. Problem 2.

3. The navigator of an oil tanker uses the satellites of the Global Positioning System (GPS/NAVSTAR) to find latitude and longitude; see Fig. 1-8. These are 43°36′25.3″ N and 77°31′48.2″ W. If the accuracy of these determinations is ±0.5″, what is the uncertainty in the tanker's position measured along (*a*) a north-south line (meridian of longitude) and (*b*) an east-west line (parallel of latitude)? (*c*) Where is the tanker?

4. In October, 1707 four British warships ran aground because of error in position, sparking the effort to produce a marine clock that would be accurate to locate a position within 30 miles after sailing from England to the West Indies and back again. (*a*) What daily accuracy for such a clock would be required? (*b*) What daily accuracy in a clock is required to fix position to within 0.5 miles after 1 year at sea? (See *Longitude* by Dava Sobel, Penguin, Baltimore, 1995.)

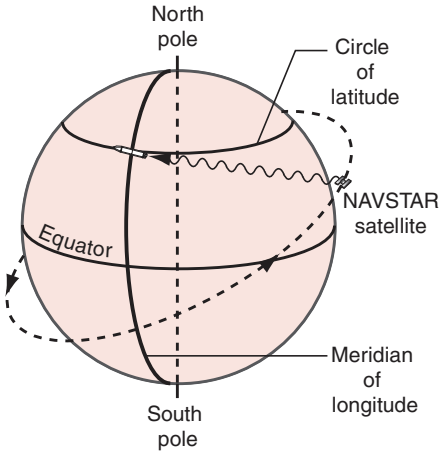

FIGURE 1-8. Problem 3.

5. During the night each breath in contains about 0.3 L of oxygen (O_2, 1.43 g/L at room temperature and pressure). Each breath out contains 0.3 L of carbon dioxide (CO_2, 1.96 g/L at room temperature and pressure). In the course of an 8-hour sleep how much weight in pounds is lost from breathing?

6. Suppose that it takes 12 h to drain a container of 5700 m^3 of water. What is the mass flow rate (in kg/s) of water from the container? The density of water is 1000 kg/m^3.

7. The grains of fine California beach sand have an average radius of 50 μm. What mass of sand grains would have a total surface area equal to the surface area of a cube exactly 1 m on an edge? Sand is made of silicon dioxide, 1 m^3 of which has a mass of 2600 kg.

8. The standard kilogram (see Fig. 1-4) is in the shape of a circular cylinder with its height equal to its diameter. Show that, for a circular cylinder of fixed volume, this equality gives the smallest surface area, thus minimizing the effects of surface contamination and wear.

9. The distance between neighboring atoms, or molecules, in a solid substance can be estimated by calculating twice the radius of a sphere with volume equal to the volume per atom of the material. Calculate the distance between neighboring atoms in (*a*) iron and (*b*) sodium. The densities of iron and sodium are 7870 kg/m^3 and 1013 kg/m^3, respectively; the mass of an iron atom is 9.27×10^{-26} kg, and the mass of a sodium atom is 3.82×10^{-26} kg.

10. (*a*) A rectangular metal plate has a length of 8.43 cm and a width of 5.12 cm. Calculate the area of the plate to the correct number of significant figures. (*b*) A circular metal plate has a radius of 3.7 cm. Calculate the area of the plate to the correct number of significant figures.